Ice Cream balls

An ice cream cone contains three balls. Find the relation between the volumes of the ice cream balls X, Y and Z.

As in the figure, P, Q and R are the centers of the ice cream balls.

B, D and E are the points of contacts of the balls with the side of the cone.

Let the radii of the balls be a = BP, b = DQ, c = ER

Draw :   QF $\perp$ BP , RG $\perp$ DQ .

Note that   BA // GR // FQ   and   BP // DQ // ER .

Then $\Delta PQF \sim \Delta QRG$ (AAA).

$\frac{PQ}{PF} = \frac{QR}{QG}$ (sides of similar Δs in ratio)

$\frac{a+b}{a-b} = \frac{b+c}{b-c}$

$(a + b)(b - c) = (b + c)(a - b)$
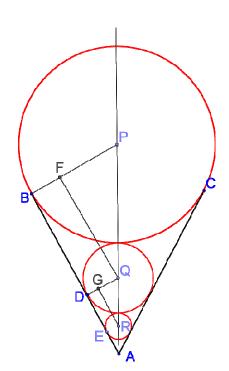$a\,b - a\,c + b^2 - b\,c = a\,b + a\,c - b^2 - b\,c$
$2b^2 = 2ac$
$b^2 = ac$
$(b^2)^3 = (ac)^3$
$(b^3)(b^3) = (a^3)(c^3)$

$\left(\frac{4}{3}\pi b^3\right)\left(\frac{4}{3}\pi b^3\right) = \left(\frac{4}{3}\pi a^3\right)\left(\frac{4}{3}\pi c^3\right)$

Since   X, Y and Z are the volumes of the balls,   (X > Y > Z) we have the relation: $Y^2 = XZ$

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